

## Some Issues in Noncommutative Solitons as $D$ -branes

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### Abstract

We investigate and interpret a large class of soliton solutions found in noncommutative tachyon condensation. These constructions make extensive use of the idea that  $Dp$ -branes may be built out of lower dimensional branes. Finally we comment on a recently proposed solution generating technique.

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# 1 Introduction

Sen has conjectured that the tachyonic vacuum in open bosonic string theory on a D-brane describes the closed string vacuum without D-branes, and that various soliton solutions in this theory describe D-branes of lower dimension[1]. Recent developments in noncommutative geometry [2, 3, 4, 5] and in particular [6] have been tied in with this conclusion, resulting in a suprisingly solvable model of D-brane annihilation. The first steps in this recent progress were made in the limit of an infinite background  $B$ -field [7, 8, 9]. Subsequently a background independent formulation of the model was proposed [10] following the reformulation of the original argument in [11, 12]. This incorporated the advantages of the infinite  $B$  limit, while also offering some insight into the vacuum structure of the closed string and connections to M(atric) theory. Though much progress has been made, a number of unresolved issues remain. These include various vacuum degeneracies, inexplicable solitonic configurations, the missing magnetic 21-brane soliton, and unwanted fluctuations on “good” solitonic solutions. Speculations on the resolution of some of these issues have been put forward [13, 14, 15]. In addition, a technique for generating solutions to the equations of motion from known solutions has been proposed [16].

In this paper we wish to expand on the idea [10] that these issues may be resolved by identifying the underlying degrees of freedom as  $D0$ -branes or  $D$ -instantons. Though this is obviously closely related to a M(atric) theory [17] interpretation, we will be working with the unstable branes of open bosonic string theory. Unlike the BPS  $D0$ -brane partons of M(atric) theory, our fundamental constituents of all  $Dp$ -branes will themselves be the unstable  $D0$ -branes. The instabilities of all of these  $Dp$ -branes are reflected in the presence of a tachyonic open string state.

The paper is organized as follows. In section 2 we write down the effective Lagrangian governing the lowest modes in open string field theory. Discussion then turns in section 3 to the issue of soliton tensions and how they essentially arise with the appearance of constituent descriptions of  $Dp$ -branes. Section 4 presents the equations of motion arising from the effective Lagrangian and proposes a simple set of conditions for identifying solutions. The main discussion in section 5 identifies both standard solutions to the equations of motion and “spurious” ones, and motivates identifications of each with underlying string/brane configurations. Section 6 considers solutions arising from an alternative form of the effective Lagrangian that has been proposed by several authors [18, 19, 20]. And the last section discusses the recently proposed solution generating technique [16].

## 2 The Effective Lagrangian

We begin with an expression for the effective Lagrangian on a single  $Dp$ -brane in bosonic string theory after having integrated out all modes except the tachyon and massless  $U(1)$  gauge field

$$L(t) = \frac{1}{(2\pi)^{\frac{p-1}{2}}} \frac{1}{g_s} \int d^p x [V(T) \sqrt{\det(g_{\mu\nu} + F_{\mu\nu})} - f(T) \partial_\mu T \partial^\mu T \sqrt{\det g_{\mu\nu}} + \dots]. \quad (2.1)$$

We have set  $2\pi\alpha' = 1$  and normalized  $V(T)$  such that  $V(T_{max}) = 1$ . With this normalization the coefficient in front of the action is exactly the  $D25$ -brane tension. The closed string background is defined by the closed string metric  $g_{\mu\nu}$ , the closed string coupling  $g_s$ , and the constant background  $B$ -field. To write the effective Lagrangian in the presence of a background  $B$ -field, we need only begin with the effective Lagrangian without a  $B$ -field (2.1), replace the closed string parameters  $g_{\mu\nu}$

and  $g_s$  by the corresponding open string parameters  $G_{\mu\nu}$  and  $G_s^2$ , and further replace all products of fields by the associative  $\star$ -product of fields defined in terms of the noncommutativity parameter  $\Theta^{\mu\nu}$  [5]. Whereas in (2.1) the tachyon is neutral under the  $U(1)$  (as evidenced by the presence of ordinary derivatives in the tachyon kinetic terms), the noncommutativity induces a nonzero coupling of the tachyon to the gauge field. Hence the ordinary derivatives of (2.1) should be replaced by gauge covariant ones. An additional freedom in writing the effective Lagrangian arises from the choice of world-sheet regularization which may be represented by a two form  $\Phi_{\mu\nu}$ . Putting these together we have

$$L(t) = \frac{1}{(2\pi)^{\frac{p-1}{2}}} \frac{1}{G_s^2} \int d^p x [V(T) \sqrt{\det(G_{\mu\nu} + F_{\mu\nu}^{NC} + \Phi_{\mu\nu})} - f(T) D_\mu T D^\mu T \sqrt{\det G_{\mu\nu}} + \dots]_\star. \quad (2.2)$$

Upon choosing a particular  $\Phi_{\mu\nu}$ , we may then express  $G_{\mu\nu}$ ,  $G_s$ , and  $\Theta^{\mu\nu}$  in terms of  $g_{\mu\nu}$ ,  $g_s$ ,  $B_{\mu\nu}$ , and  $\Phi_{\mu\nu}$  using the relations

$$\begin{aligned} \left(\frac{1}{G + \Phi}\right)^{\mu\nu} + \Theta^{\mu\nu} &= \left(\frac{1}{g + B}\right)^{\mu\nu} \\ G_s^2 &= g_s \sqrt{\frac{\det(G + B)_{\mu\nu}}{\det(g + B)_{\mu\nu}}} \end{aligned} \quad (2.3)$$

Earlier work on the subject [8, 11] used the gauge  $\Phi_{\mu\nu} = 0$ . We instead follow [10] and take  $\Phi_{\mu\nu} = -B_{\mu\nu}$  which leads to the exact expressions

$$\begin{aligned} \Phi_{\mu\nu} &= -B_{\mu\nu} \\ \Theta^{\mu\nu} &= \left(\frac{1}{B}\right)^{\mu\nu} \\ G_{\mu\nu} &= -\left(B \frac{1}{g} B\right)_{\mu\nu} \\ G_s^2 &= g_s \sqrt{\frac{\det B_{\mu\nu}}{\det g_{\mu\nu}}} \end{aligned} \quad (2.4)$$

These relations are precisely those found in the  $\alpha' B_{\mu\nu} \rightarrow \infty$  limit of the description in terms of  $\Phi_{\mu\nu} = 0$ . In order to obtain manifest background independence, we use as our gauge degrees of freedom the  $X$  fields

$$X^\mu = x^\mu + \Theta^{\mu\nu} A_\nu^{NC} \quad (2.5)$$

where the coordinates  $x^\mu$  satisfy  $[x^\mu, x^\nu] = \Theta^{\mu\nu}$ . The effective Lagrangian for a Dp-brane is then given by

$$L(t) = \frac{1}{(2\pi)^{\frac{p-1}{2}}} \frac{1}{g_s} \int \frac{d^p x}{\sqrt{\det \Theta^{\mu\nu}}} [V(T) \sqrt{\det(\delta_\mu^\nu + g_{\mu\lambda} [X^\lambda, X^\nu])} - f(T) g_{\mu\nu} [X^\mu, T] [X^\nu, T] + \dots]. \quad (2.6)$$

### 3 Tensions of Solitons

Utilizing ideas from [6] the authors of [7, 8] constructed noncommutative solitons on the worldvolume of a bosonic  $D25$ -brane and the work of Harvey et al [8] in fact identified them with  $Dp$ -branes

of higher even codimension. Part of the evidence for this identification consisted of showing that the lower dimension solitons exhibited tensions in agreement with the results for  $Dp$ -branes obtained by T-duality. Obtaining the correct tension though strongly suggestive, does not by itself constitute a complete demonstration that the solitonic configurations are the D-branes. This is because the reason that the tension comes out right has to do with the formula that gives the correspondence between functions of noncommuting coordinates and operators on a Hilbert space. In particular, for  $\Theta^{\mu\nu}$  of rank  $n$ , we may group the coordinates into  $n$  noncommuting pairs with  $26 - 2n$  leftover commuting coordinates. Functions of the 26 coordinates can then be mapped to matrix-valued functions of the  $26 - 2n$  commuting coordinates. The  $\star$ -product gets mapped to the tensor product of operator multiplication with ordinary multiplication, and most importantly the measure of integration over the noncommutative coordinates gets mapped to a trace over the Hilbert space

$$\int \frac{d^n x}{\sqrt{\det \Theta^{\mu\nu}}} \dots \rightarrow (2\pi)^{\frac{n}{2}} \text{Tr} \dots \quad (3.1)$$

Considering the Lagrangian (2.6) for  $p$ -even. We can consider turning on a  $B$ -field in  $p$  directions giving rise to a  $\Theta^{\mu\nu}$  of rank  $\frac{p}{2}$ . Using the correspondence above (and restoring factors of  $2\pi\alpha'$ ) we would then have

$$L_p(t) = \frac{1}{(2\pi)^p (\alpha')^{\frac{p+1}{2}}} \frac{1}{g_s} \int \frac{d^p x}{\sqrt{\det \Theta^{\mu\nu}}} (\dots) \longrightarrow \frac{1}{(2\pi)^p (\alpha')^{\frac{p+1}{2}}} \frac{1}{g_s} (2\pi)^{\frac{p}{2}} (2\pi\alpha')^{\frac{p}{2}} \text{Tr}(\dots) = \frac{1}{g_s \sqrt{\alpha'}} \text{Tr}(\dots) \quad (3.2)$$

which may be identified with the Lagrangian for  $N \rightarrow \infty$  D0-branes.

For  $p$ -odd, we can go to Euclidean space and consider turning on a  $B$ -field in  $p + 1$  directions. Using the operator correspondence, this leads to

$$S_p = \frac{1}{(2\pi)^p (\alpha')^{\frac{p+1}{2}}} \frac{1}{g_s} \int \frac{dtd^p x}{\sqrt{\det \Theta^{\mu\nu}}} (\dots) \longrightarrow \frac{1}{(2\pi)^p (\alpha')^{\frac{p+1}{2}}} \frac{1}{g_s} (2\pi)^{\frac{p+1}{2}} (2\pi\alpha')^{\frac{p+1}{2}} \text{Tr}(\dots) = \frac{2\pi}{g_s} \text{Tr}(\dots) \quad (3.3)$$

which may be identified with the Euclidean action for  $N \rightarrow \infty$  D-instantons. It should be obvious by these identifications that obtaining the correct tension for a soliton to be identified with a  $Dp$ -brane is built into the formalism of describing functions of noncommuting coordinates by operators on a Hilbert space. The operator description is equivalent to a constituent description. The process of selecting a solitonic profile is merely to select a subset of an infinitely extended collection of these constituent elements. In light of this, in order to properly identify a configuration its fluctuation spectrum should also be considered.

## 4 Equations of Motion

To connect with issues raised in [8, 11, 10] we consider the effective action (2.6) for  $p=25$ . For the explicit construction of codimension-2n solitons it is only necessary to turn on a background  $B$ -field in 2n directions[8]. We will, however, be utilizing a maximal rank  $B$ -field. In doing so we will enable ourselves to deal with a more general set of solutions than in [8]. To do so we work in Euclidean space. Turning on the  $B$ -field in all 26 directions and using the operator correspondence outlined in the previous section we have

$$S_{25} = \frac{2\pi}{g_s} \text{Tr} [V(\hat{T}) \sqrt{\det(\delta_\mu^\nu + g_{\mu\lambda} [\hat{X}^\lambda, \hat{X}^\nu])} - f(\hat{T}) g_{\mu\nu} [\hat{X}^\mu, \hat{T}] [\hat{X}^\nu, \hat{T}] + \dots]. \quad (4.1)$$

Defining

$$M_\mu^\nu \equiv \delta_\mu^\nu + g_{\mu\lambda}[\hat{X}^\lambda, \hat{X}^\nu] \quad (4.2)$$

the tachyon equation of motion arising from (4.1) is given by

$$V'(\hat{T})\sqrt{\det M_\mu^\nu} - f'(\hat{T})g_{\mu\nu}[\hat{X}^\mu, \hat{T}][\hat{X}^\nu, \hat{T}] + g_{\mu\nu}[\hat{X}^\mu, [\hat{X}^\nu, \hat{T}]f(\hat{T})] + g_{\mu\nu}[\hat{X}^\nu, f(\hat{T})[\hat{X}^\mu, \hat{T}]] = 0 \quad (4.3)$$

while the equation of motion from varying the  $X$  field is

$$-\frac{1}{2}[\hat{X}_\mu, (M^{-1} - (M^T)^{-1})^{\mu\nu}\sqrt{\det M_\mu^\nu}V(\hat{T})] + [\hat{T}, [\hat{X}^\nu, \hat{T}]f(\hat{T})] + [\hat{T}, f(\hat{T})[\hat{X}^\nu, \hat{T}]] = 0 \quad (4.4)$$

To simplify matters we may consider a sufficient, but perhaps not necessary, set of conditions which will lead to solutions  $\hat{X}_c$  and  $\hat{T}_c$  of the equations above

$$\begin{aligned} a. \quad & V'(\hat{T}) = 0 \\ b. \quad & [\hat{X}^\mu, \hat{T}] = 0 \\ c. \quad & [\hat{X}^\mu, [\hat{X}^\nu, \hat{X}^\lambda]] = 0 \end{aligned} \quad (4.5)$$

These conditions may not admit the complete set of solutions to the equations of motion (4.3,4.4), but we leave this additional complication for future work.

## 5 Solutions

We now consider solutions to these equations. The first three represent configurations that have a definite interpretation in terms of standard string/brane configurations [7, 8, 11, 10]. The remaining solutions have less obvious interpretations, and as such must either be accounted for in standard string/brane configurations or somehow excluded in this context. One should keep in mind that the operator correspondence maps the gauge covariant derivative in a particular direction to a commutator term involving the  $X$  field

$$D_\mu \rightarrow -i\Theta_{\mu\nu}^{-1}[\hat{X}^\nu, \quad], \quad (5.1)$$

so that if we have a background solution  $\hat{X}^\mu = \lambda\hat{I}$ , then propagating fluctuations in the  $x^\mu$  direction are forbidden.

$$\hat{T}_c = T_{max}\hat{I} \quad \hat{X}_c^\mu = \hat{x}^\mu \quad \text{where} \quad [\hat{x}^\mu, \hat{x}^\nu] = \Theta^{\mu\nu}\hat{I}. \quad (5.2)$$

This solution represents a uniform open string tachyon field on the worldvolume of an unstable  $D25$ -brane. Fluctuations about this background form a noncommutative  $U(1)$  gauge theory with 26-dimensional tachyons transforming in the adjoint. As a symmetry among the operators, the noncommutative  $U(1)$  is realized as a  $\bigotimes_{i=1}^{13} U(N_i \rightarrow \infty)$ . The tension for this configuration may be identified by inserting the background field configurations into the action;

$$S_{background} = \frac{1}{g_s} \frac{1}{(2\pi)^{\frac{25-1}{2}}} \int \frac{d^{26}x}{\sqrt{\det \Theta}} V(T_{max}) \sqrt{\det(\delta_\mu^\nu + g_{\mu\lambda}\Theta^{\lambda\nu})}. \quad (5.3)$$

For  $V(T_{max}) = 1$ , we identify the coefficient of the integral over the 26-dimensional worldvolume as the tension of the  $D25$ -brane.

$$\hat{T}_c = T_{min} \hat{I} \quad \hat{X}_c^\mu = \lambda \hat{I} \quad (5.4)$$

By Sen's conjecture, this uniform solution represents the stable closed string vacuum in the absence of  $D$ -branes. There are no propagating open string tachyon fluctuations in this background. The action with this background becomes

$$S_{background} = \frac{1}{g_s} \frac{1}{(2\pi)^{\frac{25-1}{2}}} \int \frac{d^{26}x}{\sqrt{\det \Theta}} V(T_{min}) \sqrt{\det(\delta_\mu^\nu)} \quad (5.5)$$

which vanishes according to Sen's conjecture, i.e.  $V(T_{min}) = 0$ .

$$\hat{T}_c = T_{max} \hat{P}_n + T_{min} (\hat{I} - \hat{P}_n) \quad \hat{X}_c^i = \hat{x}^i \hat{P}_n \quad i = 0, \dots, p \quad \hat{X}_c^m = \lambda \hat{I} \quad m = p+1, \dots, 25 \quad (5.6)$$

The  $\hat{P}_n$  in this expression are projection operators onto the Hilbert space generated by the "transverse" noncommuting coordinates  $x^m$ . The tachyon profile expressed in terms of  $\hat{P}_n$  enjoys the useful property that

$$V(\hat{T}_c) = V(T_{max}) \hat{P}_n + V(T_{min}) (\hat{I} - \hat{P}_n) \quad (5.7)$$

These noncommutative solitons have finite extent in the  $x^m$  directions and infinite extent in the  $x^i$  directions. Such backgrounds interpolate in the transverse directions  $x^m$  between the tachyonic vacuum in the core of the soliton and the closed string vacuum outside of the soliton. Convincing evidence has been put forward to identify these configurations with lower dimensional unstable  $Dp$ -branes[7, 8, 11]. To clarify this picture we may consider a block diagonal noncommutativity matrix

$$\Theta = \bigoplus_{k=1}^{13} \begin{pmatrix} o & \theta_k \\ -\theta_k & o \end{pmatrix} \quad (5.8)$$

where

$$[x^{2k}, x^{2k+1}] = i\theta_k. \quad (5.9)$$

Rank  $n_k$  projection operators  $P_{n_k}$  can be constructed on the Hilbert spaces  $H_k$  formed from each noncommuting pair of coordinates, so that a general projection operator takes the form

$$P_n = P_{n_1}^{(1)} \otimes P_{n_2}^{(2)} \otimes \dots \otimes P_{n_{\frac{25-p}{2}}}^{(\frac{25-p}{2})} \quad (5.10)$$

for  $p$  odd. A soliton of this form corresponds to the chain of decays

$$1D25 \rightarrow n_1 D23 \rightarrow n_1 n_2 D21 \rightarrow \dots \rightarrow \prod_{k=1}^{\frac{25-p}{2}} n_k Dp \quad (5.11)$$

To concretely identify this configuration with  $n$  coincident  $Dp$ -branes, we insert the solution back into (4.1) and use the correspondence (3.1) on  $p+1$  of the coordinates to obtain

$$S_{background} = \frac{1}{g_s} \frac{1}{(2\pi)^{\frac{p-1}{2}}} Tr[\hat{P}_n V(T_{max}) + (\hat{I} - \hat{P}_n) V(T_{min})] \int \frac{d^{p+1}x}{\sqrt{\det \Theta'}} \sqrt{\det(\delta_i^j + g_{ik} \Theta'^{kj})} \quad (5.12)$$

where

$$\Theta' = \bigoplus_{k=1}^{\frac{p+1}{2}} \begin{pmatrix} o & \theta_k \\ -\theta_k & o \end{pmatrix} \quad (5.13)$$

If we use  $V(T_{max}) = 1$  and Sen's conjecture, i.e.  $V(T_{min}) = 0$ , then (5.12) becomes

$$S_{background} = \frac{n}{g_s} \frac{1}{(2\pi)^{\frac{p-1}{2}}} \int \frac{d^{p+1}x}{\sqrt{\det\Theta'}} \sqrt{\det(\delta_i^j + g_{ik}\Theta'^{kj})} \quad (5.14)$$

which if compared to (5.3) is easily identified as the action for  $n$   $Dp$ -branes.

It should be noted that one can take the limit  $\theta_k \rightarrow 0$  in both (5.3) and (5.14) after redefining the coordinates  $x_{2k}, 2k+1 \rightarrow \frac{x_{2k, 2k+1}}{\sqrt{\theta_k}}$  thus recovering the usual form of the D-brane action in the absence of gauge fields or  $B$  fields. Thus our procedure using the background independent formalism of [10] avoids the ambiguities associated with taking a large  $B$  limit.

The solutions above admit simple and elegant interpretations in terms of coincident unstable  $Dp$ -branes and the closed string vacuum. However, other nonsingular solutions to (4.5) exist, and thus solve the equations of motion arising from (4.1). Since the action 2.1 is expected to be an approximation to the complete string field theory in the limit where the derivatives of the gauge fields and  $B$  fields are small, then these solutions must also be accounted for as configurations of perturbative and nonperturbative states in string theory. The known perturbative and nonperturbative states of bosonic string theory include the fundamental string and its magnetic dual, as well as unstable  $Dp$ -branes for  $p = -1, \dots, 25$ . However, the string field theory action, if complete, could not only predict these states, but any possible configuration of these states consistent with the background in which the string field theory is formulated. Certainly a number of the smooth solutions to the full action will arise from its low energy effective form. It is via these nontrivial configurations that we will interpret the additional solutions. In our case this background is the worldvolume of an unstable  $D25$ -brane with a constant maximal  $B$ -field.

A set of additional solutions to (4.5) was first pointed out in [11]. These involve allowing different projection operators to define the transverse profiles of the tachyon and  $X$  fields. To systematically cover this set of solutions we will consider descending from the  $D25$ -brane configuration by replacing the identity operator by projectors where appropriate. We first investigate the effect of nontrivial projection operators for the tachyon while maintaining trivial forms for  $\hat{X}^L, \hat{X}^T$ . We then study the effects on  $\hat{X}^L$ . These results may be combined for configurations with nontrivial projection operators for both  $\hat{T}$  and  $\hat{X}^L$ .

For now we consider operators projecting onto the subspace generated by a single pair of coordinates which we will refer to as simply  $\hat{x}^T$ , since these will in some sense be interpreted as directions transverse to the resulting system. The remaining coordinates we refer to as  $\hat{x}^L$  since these will be roughly longitudinal to the system. We consider functions of the coordinates  $\vartheta(x^\mu)$  which may be represented by direct product operators  $\hat{\vartheta} = \hat{\vartheta}_T \otimes \hat{\vartheta}_L$ . So we have for the identity on the entire Hilbert space  $H = H_L \otimes H_T$  an expression  $\hat{I}_{L,T} = \hat{I}_L \otimes \hat{I}_T$ , and for a rank  $n$  projection operator  $\hat{P}_n = \hat{I}_L \otimes \hat{P}_{nT}$ .

Consider a set of operators split into longitudinal and transverse parts. The set of solutions takes the form

$$\begin{aligned} \hat{T}_c &= T_{max} \hat{I}_L \otimes \hat{P}_{n1T} + T_{min} \hat{I}_L \otimes (\hat{I}_T - \hat{P}_{n1T}) \\ \hat{X}_c^L &= \hat{x}^L \otimes \hat{P}_{n2T} \end{aligned} \quad (5.15)$$

The form of the transverse  $X$  field determines two branches in the space of solutions

$$\begin{aligned} 1. \quad \hat{X}_c^T &= \hat{I}_L \otimes \hat{x}^T \quad \text{with} \quad \hat{P}_{n1T}, \hat{P}_{n2T} = \hat{0}^T \text{ or } \hat{I}_T \\ 2. \quad \hat{X}_c^T &= \lambda \hat{I}_L \otimes \hat{I}_T \quad \text{with} \quad [\hat{P}_{n1T}, \hat{P}_{n2T}] = 0. \end{aligned} \quad (5.16)$$

We begin by descending the tachyon profile from  $\hat{I}_{L,T} \rightarrow \hat{I}_L \otimes \hat{P}_{nT} \rightarrow \hat{I}_L \otimes \hat{0}_T = \hat{0}_{L,T}$  while keeping in mind (5.16) and holding  $\hat{X}^L$  fixed

$$\begin{array}{lll} a) & \hat{T}_c = T_{max} \hat{I}_{L,T} & \hat{X}_c^L = \hat{x}^L \otimes \hat{I}_T \quad \hat{X}_c^T = \hat{I}_L \otimes \hat{x}^T \\ & \downarrow & \downarrow \\ b) & \hat{T}_c = T_{max} \hat{I}_{L,T} & \hat{X}_c^L = \hat{x}^L \otimes \hat{I}_T \quad \hat{X}_c^T = \lambda \hat{I}_L \otimes \hat{I}_T \\ & \downarrow & \downarrow \\ c) & \hat{T}_c = T_{max} \hat{I}_L \otimes \hat{P}_{nT} + T_{min} \hat{I}_L \otimes (\hat{I}_T - \hat{P}_{nT}) & \hat{X}_c^L = \hat{x}^L \otimes \hat{I}_T \quad \hat{X}_c^T = \lambda \hat{I}_L \otimes \hat{I}_T \\ & \downarrow & \downarrow \\ d) & \hat{T}_c = T_{min} \hat{I}_{L,T} & \hat{X}_c^L = \hat{x}^L \otimes \hat{I}_T \quad \hat{X}_c^T = \lambda \hat{I}_L \otimes \hat{I}_T \\ & \downarrow & \\ e) & \hat{T}_c = T_{min} \hat{I}_{L,T} & \hat{X}_c^L = \hat{x}^L \otimes \hat{I}_T \quad \hat{X}_c^T = \hat{I}_L \otimes \hat{x}^T \end{array} \quad (5.17)$$

One should keep in mind that the tachyon profile will always lead to a simple tension expression which can then be used to identify the  $Dp$ -brane present. The  $X$  field configuration on the other hand governs the propagation of various fluctuations, and so should give us information on how the constituent  $Dp$ -branes are assembled.

The process  $a \rightarrow b$  represents a transition from a space filling  $D25$ -brane into a space filling stack in the  $x^T$  directions of an infinite number of  $D23$ -branes with worldvolume extension in the  $x^L$  directions, and with open strings confined to each constituent brane. That open string modes cannot propagate in  $x^T$  is a consequence of the vanishing of the covariant derivative in the transverse directions

$$D_T \rightarrow -i\Theta_{\mu\nu}^{-1}[\lambda \hat{I}_L \otimes \hat{I}_L, \quad ] = 0 \quad (5.18)$$

In essence, propagation in the  $x^T$  directions is eliminated by not allowing open strings to migrate from one  $D23$ -brane to the next. Propagation of fluctuations along the  $D23$ -branes is of course still allowed.

The transition  $b \rightarrow c$  represents the decay of  $\infty - n$  of the  $D23$ -branes into the closed string vacuum. In contrast to the standard  $D23$ -brane solution which implements the same projection operator for the tachyon and longitudinal  $X$  fields, here we have allowed a nontrivial projection operator for the tachyon alone. However, the resulting configuration is physically indistinguishable from the standard  $D23$ -brane solution (5.6). This is a simple consequence of the factors of  $V(T)$  and  $f(T)$  in front of the Born-Infeld and tachyon kinetic terms respectively. These give rise to an overall factor of the tachyon projection operator which acts on the  $X$  fields, effectively giving  $\hat{X}^L$  the same projector form as  $\hat{T}$ . The action evaluated for this background is precisely (5.12). In this case (with the projector profile) we can not reinstate propagation in the  $x^T$  directions in light of (5.16). This is a reflection of the simple fact that a  $D25$ -brane cannot be constructed out of a finite number of  $D23$ -branes.

The transition  $c \rightarrow d$  seems to have no definite interpretation unless the assumption  $f(T_{min}) = 0$  is made. With this assumption configuration  $d$  is physically identical to the closed string vacuum



(5.4). Without this assumption the tachyon profile will yield a vanishing tension, yet propagating fluctuations are allowed along the  $x^L$  directions. One should note however that with this choice for the tachyon field, the overall coefficient  $V(T_{min})$  of the Born-Infeld contribution to the action vanishes. We expect that our computations, based on a well defined action, will run into trouble in this scenario.

The transition  $a \rightarrow e$  shares the complication of  $a \rightarrow b$  and will need further investigation of the action to be interpreted or excluded. Sen has argued [14] that configurations  $d$  and  $e$  are actually equivalent descriptions of the closed string vacuum.

We may now consider the results of descending the longitudinal  $X$  field  $\hat{X}^L$ , this time holding fixed the tachyon while maintaining (5.16).

$$\begin{array}{lll}
a) \quad \hat{T}_c = T_{max} \hat{I}_{L,T} & \hat{X}_c^L = \hat{x}^L \otimes \hat{I}_T & \hat{X}_c^T = \hat{I}_L \otimes \hat{x}^T \\
& \downarrow & \downarrow \\
b) \quad \hat{T}_c = T_{max} \hat{I}_{L,T} & \hat{X}_c^L = \hat{x}^L \otimes \hat{I}_T & \hat{X}_c^T = \lambda \hat{I}_L \otimes \hat{I}_T \\
& \downarrow & \downarrow \\
c) \quad \hat{T}_c = T_{max} \hat{I}_{L,T} & \hat{X}_c^L = \hat{x}^L \otimes \hat{P}_{nT} & \hat{X}_c^T = \lambda \hat{I}_L \otimes \hat{I}_T \\
& \downarrow & \downarrow \\
d) \quad \hat{T}_c = T_{max} \hat{I}_{L,T} & \hat{X}_c^L = \hat{I}_L \otimes \hat{0}^T \cong \lambda \hat{I}_L \otimes \hat{I}_T & \hat{X}_c^T = \lambda \hat{I}_L \otimes \hat{I}_T \\
& & \downarrow \\
e) \quad \hat{T}_c = T_{max} \hat{I}_{L,T} & \hat{X}_c^L = \hat{I}_L \otimes \hat{0}^T \cong \lambda \hat{I}_L \otimes \hat{I}_T & \hat{X}_c^T = \hat{I}_L \otimes \hat{x}^T
\end{array} \tag{5.19}$$

We identify in  $c, d$  that taking the longitudinal  $X$  field projection operator to have rank 0 is effectively equivalent to setting  $\hat{X}^L \propto \hat{I}$ .

The process  $a \rightarrow b$  is identical to the case discussed for (5.17), that is the transition from a  $D25$ -brane into an infinite number of  $D23$ -branes with open string ends restricted to single  $D23$ -branes.

The transition  $b \rightarrow c$  is rather interesting. The tension is that of the space filling  $D25$ -brane. The propagation of fluctuations in the transverse directions  $x^T$  is eliminated as in configuration  $b$ . However, now the propagation of fluctuations in the longitudinal directions  $x^L$  is restricted to a region localized around the origin in  $x^T$  as described by  $\hat{P}_{nT}$ . The interpretation of this is as follows: each of  $\infty - n$  of the infinite collection of  $D23$ -branes in configuration  $b$  undergoes a transition to an infinite collection of  $D$ -instantons with the ends of any open string confined to a single  $D$ -instanton. This confinement eliminates propagating fluctuations in the longitudinal directions  $x^L$  outside of a region localized in  $x^T$ . The remaining  $nD23$ -branes are stacked as before about the origin in  $x^T$  and admit propagating fluctuations along  $x^L$ .

The transition  $c \rightarrow d$  corresponds to a transition of the remaining  $D23$ -branes in configuration  $c$  to infinite collections of  $D$ -instantons. Since the tachyon profile has not changed, we expect an energy density identical to that of a  $D25$ -brane. In fact the action evaluated with this background field configuration is essentially that of (3.3), i.e.

$$S_{background} = \frac{2\pi}{g_s} Tr \hat{I}_{L,T} = \frac{2\pi}{g_s} (N \rightarrow \infty)^{13} \tag{5.20}$$

Since the ends of open strings are confined to points in space-time, there are of course no propagating open string modes.

The transition  $a \rightarrow e$  is actually very closely related to the  $a \rightarrow b$  transition. In this case it may seem that the transition of the space filling  $D25$ -brane is to an infinite collection of  $D1$ -branes. However, in interpreting configuration  $c$  which is extended in the  $x^L$  directions as a stack of

coincident  $D23$ -branes, we are assuming that time is an element of the  $x^L$  set of coordinates. Thus for configuration  $e$  we have a stack of two-dimensional objects extended in purely spatial directions  $x^T$  and exist only at an instant in time. These extended instantons correspond to objects resulting from  $T$ -dualizing the time coordinate on a  $D2$ -brane. Whether this operation is well defined and the role played by the resulting objects is outside the scope of this paper. For a discussion of such issues see [22] and references therein.

The two descent chains above may be used as the cornerstones for more complicated projector combinations. In any case one is led to a description of the  $D25$ -brane in terms of some constituent  $Dp$ -branes which are either decayed, transversely distributed, coincident, or some combination of these which may differ for different directions.

A final set of solutions that may be considered are those which utilize operators on the transverse subspace other than  $\hat{I}, \hat{P}_n, \hat{0}, \hat{x}^T$ . These must commute with any other operator acting in this subspace in order to serve as solutions to (4.5). The operators above are distinguished by possessing eigenvalues equal to either 0 or 1, for  $\hat{I}, \hat{P}_n, \hat{0}$  and values filling out the real line for  $\hat{x}^T$ . Commuting operators with more diverse spectra of eigenvalues certainly exist. To this end we may consider solutions of the form

$$\hat{T}_c = T_{max} \hat{I}_L \otimes \hat{P}_{n1T} + T_{min} \hat{I}_L \otimes (\hat{I}_T - \hat{P}_{n1T}) \quad \hat{X}_c^L = \hat{x}^L \otimes \hat{M} \quad \hat{X}_c^T = 0 \quad (5.21)$$

where

$$\hat{M} \equiv \sum_{i=1}^k \lambda_i \hat{P}_{n_i}^i \quad \text{for} \quad \hat{P}_{n_i}^i \hat{P}_{n_j}^j = \hat{P}_{n_i}^i \delta^{ij} \quad \text{and} \quad [\hat{M}, \hat{P}_{n1T}] = 0 \quad (5.22)$$

We may simplify matters by assuming

$$Tr \hat{M} = Tr \hat{P}_{n1T} \quad \hat{M} \hat{P}_{n1T} = \hat{M}. \quad (5.23)$$

to avoid the complications discussed after equation (5.15).

For all  $\lambda_i$  distinct, this configuration maintains only a  $\bigotimes_{i=1}^k U(n_i)$  of the  $U(\sum_{i=1}^k n_i)$  present in the most degenerate case. From the gauge theory point of view this symmetry breaking corresponds to a separation of the branes. Interpreting the separation in eigenvalue space as a spacetime distance à la M(atric) theory [17], this configuration can be identified with a collection of  $k$  non-coincident stacks of  $D23$ -branes.

## 6 Alternative action

By demanding consistency with T-duality several authors [18, 19, 20] have obtained alternate forms of our starting point (2.1) which differ by including the tachyon kinetic term under the square root

$$L(t) = \frac{1}{(2\pi)^{\frac{p-1}{2}}} \frac{1}{g_s} \int d^p x V(T) \sqrt{\det(g_{\mu\nu} + F_{\mu\nu} + \partial_\mu T \partial_\nu T)}. \quad (6.1)$$

An obvious advantage in considering actions of this form is the automatic vanishing of the tachyon kinetic term for  $T = T_{min}$ . We may repeat the analysis above for this modified effective Lagrangian. Turning on a maximal rank  $B$ -field and using the operator correspondence we obtain

$$S_{25} = \frac{2\pi}{g_s} Tr V(\hat{T}) \sqrt{\det(\delta_\mu^\nu + g_{\mu\lambda}[\hat{X}^\lambda, \hat{X}^\nu] + g_{\mu\nu}[\hat{X}^\mu, \hat{T}][\hat{X}^\nu, \hat{T}])}. \quad (6.2)$$

Define

$$W \equiv \delta_\mu^\nu + g_{\mu\lambda}[\hat{X}^\lambda, \hat{X}^\nu] + g_{\mu\lambda}[\hat{X}^\lambda, \hat{T}][\hat{X}^\nu, \hat{T}]. \quad (6.3)$$

The tachyon equation of motion is now given by

$$V'(\hat{T})\sqrt{\det W_\mu^\nu} + [\hat{X}_\mu, [\hat{X}_\nu, \hat{T}]]W^{-1\mu\nu}\sqrt{\det W_\mu^\nu}V(\hat{T}) - [\hat{X}_\nu, W^{-1\mu\nu}\sqrt{\det W_\mu^\nu}V(\hat{T})][\hat{X}_\mu, \hat{T}] = 0 \quad (6.4)$$

while the equation of motion from varying the  $X$  field is

$$\begin{aligned} [\hat{X}_\mu, (W^{-1} - (W^T)^{-1})^{\nu\mu}\sqrt{\det W_\mu^\nu}V(\hat{T})] + [\hat{T}, [\hat{X}_\mu, \hat{T}]W^{-1\mu\nu}\sqrt{\det W_\mu^\nu}V(\hat{T})] \\ + [\hat{T}, W^{-1\nu\mu}\sqrt{\det W_\mu^\nu}V(\hat{T})][\hat{X}_\mu, \hat{T}] = 0. \end{aligned} \quad (6.5)$$

Again we may consider a set of sufficient conditions for a solution of these equations

$$\begin{aligned} V'(\hat{T}_c) &= 0 \\ [\hat{X}_c^\mu, [\hat{X}_c^\nu, \hat{T}_c]] &= 0 \\ [\hat{X}_c^\mu, [\hat{X}_c^\nu, \hat{X}_c^\lambda]] &= 0. \end{aligned} \quad (6.6)$$

These conditions admit all of the configurations discussed in the previous section as solutions since  $\hat{X}_c^\nu$  and  $\hat{T}_c$  satisfying

$$[\hat{X}_c^\nu, \hat{T}_c] = 0 \quad (6.7)$$

certainly describe a subset of the solutions of (6.6). However, the conditions (6.6) admit now a larger set of solutions including, for example, configurations satisfying

$$[\hat{X}_c^\nu, \hat{T}_c] \propto \hat{I}. \quad (6.8)$$

Such solutions are considerably more difficult to explicitly construct than those in the preceding discussion. However, owing to the advantage of the automatically vanishing tachyon kinetic term for  $T = T_{min}$  it would be worthwhile to investigate these solutions further.

## 7 The Shift Operation

Recently a technique was introduced to facilitate finding solutions to the equations of motion for noncommutative gauge theories from known solutions by acting with an “almost” gauge transformation [16]. This method was applied to vacuum solutions in open string field theory to obtain solitonic field configurations which might then be interpreted as  $Dp$ -branes. There are a few issues regarding this construction which we feel should be discussed.

- The shift operation

Formulation of the solution generating technique began by observing that a transformation obeying

$$\begin{aligned}\hat{U}^\dagger \hat{U} &= \hat{I} \\ \hat{U} \hat{U}^\dagger &= \hat{P}\end{aligned}\tag{7.1}$$

where  $\hat{P}$  is a projection operator, when applied to the fields  $\hat{\vartheta}^i$  in an equation of motion would result in new field configurations obeying the same equation of motion.

- Tensions and the tachyon

Solutions to (7.1) only exist for infinite dimensional  $\hat{U}$ . The authors of [16] construct an infinite dimensional representation with the shift operators

$$\hat{S} = \sum_{k=0}^{\infty} |k+1\rangle\langle k| \tag{7.2}$$

which satisfy

$$\hat{S}^{n\dagger} \hat{S}^n = I, \quad \hat{S}^n \hat{S}^{n\dagger} = \hat{I} - \hat{P}_n \tag{7.3}$$

where  $\hat{P}_n$  are projection operators onto the first  $n$  states. The effect of  $\hat{U} = \hat{S}^n$  on the matrix representation of a field  $\hat{\vartheta}$  is a “southeast shift”. The idea proposed in [16] is that by acting on the closed string vacuum field configurations with  $\hat{U}$  defined above one may generate configurations corresponding to  $Dp$ -branes. To see this in action, we will look at the effect of  $\hat{U} = \hat{S}^n$  on the vacuum tachyon field configuration discussed in section 5. We will merely consider trying to build a pair of  $D23$ -branes from the closed string vacuum, so that the corresponding projection operator is nontrivial in the subspace  $H_k$  generated by  $[\hat{x}^{24}, \hat{x}^{25}] = i\theta \hat{I}$ . The tachyon vacuum configuration transforms as follows

$$\hat{T}_{vac} = \begin{pmatrix} T_{min} & 0 & 0 & 0 & \dots \\ 0 & T_{min} & 0 & 0 & \dots \\ 0 & 0 & T_{min} & 0 & \dots \\ 0 & 0 & 0 & T_{min} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \longrightarrow \hat{S}^2 \hat{T}_{vac} \hat{S}^{2\dagger} = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & T_{min} & 0 & \dots \\ 0 & 0 & 0 & T_{min} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{7.4}$$

The resulting configuration may be identified with the tachyon configuration corresponding to a pair of  $D23$ -branes

$$\hat{T}_{D23} = \begin{pmatrix} T_{max} & 0 & 0 & 0 & \dots \\ 0 & T_{max} & 0 & 0 & \dots \\ 0 & 0 & T_{min} & 0 & \dots \\ 0 & 0 & 0 & T_{min} & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{7.5}$$

only if we have arranged that  $T_{max} = 0$ . For this mechanism to work for a choice of  $T_{max} \neq 0$ , it would be necessary for the shift to produce an upper left diagonal block  $diag(T_{max}, \dots, T_{max})$ .

However, a “shift” operation accommodating nonzero “northwest” elements can not be constructed, and so this procedure exhibits a peculiar dependance on the value of what one might have expected to be an arbitrary choice. If the choice  $T_{max} = 0$  is made, then one obtains the correct tension for the  $D23$  pair in light of (5.7).

The point is that an equation of motion of the form  $F(\vartheta) = 0$  will give rise to an equation  $\hat{U}F(\vartheta)\hat{U}^\dagger = 0$  under the action of the shift transformation. But this is not the same as  $F(\hat{U}\vartheta\hat{U}^\dagger) = 0$  unless  $F(0) = 0$  is true as well.

- $X$  fields

Let us now investigate the result of the shift transformation on the gauge field configurations corresponding to the closed string vacuum in our formalism. Applying  $\hat{U} = \hat{S}^2$  to  $\hat{X}_{vac}^\mu$  we have

$$\hat{X}_{vac}^\mu = \hat{0} \longrightarrow \hat{S}^2 \hat{X}_{vac}^\mu \hat{S}^{2\dagger} = \hat{0} \quad (7.6)$$

where  $\hat{0}$  represents the null matrix. The result above will pose a problem when we compare the shifted  $\hat{X}_{vac}^\mu$  to the expected  $X$  field configuration for a pair of  $D23$ -branes (see (5.6))

$$\hat{X}_{D23}^i = \begin{pmatrix} \hat{x}^i & 0 & 0 & 0 & \dots \\ 0 & \hat{x}^i & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad i = 0, \dots, 23 \quad \hat{X}_{D23}^m = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad m = 24, 25 \quad (7.7)$$

It appears that in order for this technique to work the shift operation would have to distinguish between components of the  $X$  field, and produce a nonzero “northwest” block for the components along the brane. In addition, the transformation would have to distinguish between the tachyon and  $X$  fields and produce appropriate “northwest” blocks for each.

This issue does not arise in [16]. In that work a choice is made for the closed string vacuum configuration which effectively reverses the situation above, that is

$$\hat{X}_{vac'}^\mu = \hat{x}^\mu. \quad (7.8)$$

$X$  field configurations for the  $D$ -branes are identified as

$$\hat{X}_{D23'}^i = \begin{pmatrix} 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & \hat{x}^i & 0 & \dots \\ 0 & 0 & 0 & \hat{x}^i & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad i = 0, 1 \quad \hat{X}_{D23'}^m = \begin{pmatrix} \hat{x}^m & 0 & 0 & 0 & \dots \\ 0 & \hat{x}^m & 0 & 0 & \dots \\ 0 & 0 & \hat{x}^m & 0 & \dots \\ 0 & 0 & 0 & \hat{x}^m & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \quad m = 2, \dots, 25 \quad (7.9)$$

Again, the solution generating transformation seems to depend on the choice of 0 for the diagonal  $X$  field terms representing  $D$ -branes. This identification arises directly from the initial choice (7.8) for the vacuum  $X$  field configurations. In [16] it is proposed that these configurations afford an extension of the  $\hat{X}^\mu = 0$  vacuum to appropriate configurations for arbitrary noncommutativity  $\theta$ . These configurations reproduce the correct expressions for

$D$ -brane tensions, however as discussed in section 3, the evidence for identifying these as the correct configurations should take into account the spectrum of fluctuations as well. The configuration (7.8) will admit a spectrum of fluctuations identifiable with that of the closed string vacuum if one of two conditions hold. Either one works in the  $\alpha'B \rightarrow \infty$  limit or one conjectures that the coefficient function for the tachyon kinetic term in the action vanishes for  $\hat{T} = T_{min}\hat{I}$ , i.e.  $f(T_{min}) = 0$ . The actions discussed in section 6 automatically enforce the latter of these.

## 8 Conclusions

The explicit construction of  $Dp$ -branes as lumps in open string field theory and the emergence of the closed string vacuum provides a strong indication that a string field formulation may provide a complete nonperturbative definition of string theory. Of course we have limited ourselves to the simple setting of the bosonic string, but complications that must be dealt with in the bosonic theory are sure to arise in the supersymmetric formulation as well.

As a nonperturbative formulation, the string field action should predict all of the known perturbative and nonperturbative states in string theory. In addition, we expect that all solutions of the string field equations of motion should find some interpretation in terms of string/brane configurations. We have demonstrated that a large class of the explicit solutions constructed via techniques from noncommutative geometry have such an interpretation. A main feature of these constructions involved viewing higher dimensional  $Dp$ -branes to be composed of lower dimensional  $Dp$ -branes. In particular, infinite configurations of  $D$ -instantons allow one to account for the tension of higher dimensional branes, while forbidding propagating fluctuations. Though the general solutions become very complicated very quickly, we expect that the simple ideas presented here can be used to construct any configuration required.

Extending the bosonic string field effective action to include modifications consistent with  $T$ -duality seems to enlarge the space of solutions. Explicit construction of these new solutions is difficult, and their interpretations will certainly not be straight forward. On the other hand an advantage to this form of the action is that the tachyon kinetic term automatically vanishes for  $T = T_{min}$ . Future work may involve analyzing and interpreting these new configurations.

The shift symmetry solution generating technique may provide some important insight into noncommutative tachyon condensation. There is certainly an appeal to the generation of solutions from solutions via a single well defined transformation. The construction is reminiscent of  $T$ -duality and it would be nice to have a better understanding of the issues discussed in section 7.

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